M.Sc. - II (Mathematics) (NEP Pattern) Semester-III

# 03NEPMATH04.1 - Elective Paper-IV : Advanced Topics in Operations Research

P. Pages : 3 | **GUG/S/25/16016**

Time : Three Hours | Max. Marks : 80

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Notes :
1. Solve all the **five** questions.
2. Each equation carries equal marks.

**UNIT – I**

**1.** a) Find the optimum integer solution to the following L. P. P. : **8**

Maximize $z = x_1 + 4x_2$

subject to the constraints :

$2x_1 + 4x_2 \le 7,\ 5x_1 + 3x_2 \le 15,\ x_1,\ x_2 \ge 0$ & are integer.

b) Find the optimum integer solution to following all I. P. P. : **8**

Maximize $z = x_1 + 2x_2$ subject to constraints ,

$x_1 + x_2 \le 7,\ 2x_1 \le 11,\ 2x_2 \le 7,\ x_1, x_2 \ge 0$ & are integers.

**OR**

c) Solve the following mixed integer programming problem: **8**

maximize $z = 4x_1 + 6x_2 + 2x_3$ subject to the constraints,

$4x_1 - 4x_2 \le 5,\ -x_1 + 6x_2 \le 5,\ -x_1 + x_2 + x_3 \le 5$

$x_1,\ x_2,\ x_3 \ge 0,\ x_1$ and $x_3$ are integers.

d) Use branch and bound method to solve the following L. P. P. : **8**

maximize $z = 7x_1 + 9x_2$ subject to the constraints :

$-x_1 + 3x_2 \le 6,\ 7x_1 + x_2 \le 35,\ x_2 \le 7,\ x_1, x_2 \ge 0$ and are integers.

**UNIT – II**

**2.** a) A firm produces two products, say X and Y. Product X sells for a net profit of Rs. 80 per unit while Y sells for a net profit of Rs. 40 per unit. The goal of the firm is to earn Rs. 900 in the next week. Also, the management want to achieve sales volume for the two products close to 17 and 15 respectively. Formulate this problem as a goal programming model. **8**

b) Use Simplex method to solve the following goal programming problem: **8**

Minimize $Z = P_1\, d_1^- + 5P_3\, d_2^- + 3P_3 d_3^- + P_2 d_4^+ + P_4 d_1^+$

subject to the constraints,

$x_1 + x_2 + d_1^- - d_1^+ = 80$

$x_1 + x_2 + d_4^- - d_4^+ = 90$

$x_1 + d_2^- = 70$

and $x_2 + d_3^- = 45$

where, $x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^-, d_4^-, d_4^+ \ge 0$ .

**OR**

c) Using the bounded variable technique, solve the following L. P. P. : **8**

Maximize $z = x_2 + 3x_3$ subject to the constraints:

$x_1 + x_2 + x_3 \le 10,\ x_1 - 2x_3 \ge 0,\ 2x_2 - x_3 \le 10,$

$0 \le x_1 \le 8,\ 0 \le x_2 \le 4,\ x_3 \ge 0.$

d) Solve the following L. P. P. : **8**

Maximize $z = \dfrac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$ subject to the constraints,

$3x_1 + 5x_2 \le 15,\ 5x_1 + 2x_2 \le 10$ and $x_1, x_2 \ge 0.$

## UNIT – III

**3.** a) Determine the optional sequence of jobs that minimize the total elapsed time based on following information processing time on machines is given in hours and passing is not allowed: **8**

| Job | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Machine $M_1$ | 3 | 8 | 7 | 4 | 9 | 8 | 7 |
| Machine $M_2$ | 4 | 3 | 2 | 5 | 1 | 4 | 3 |
| Machine $M_3$ | 6 | 7 | 5 | 11 | 5 | 6 | 12 |

b) Use graphical method to minimize the time added to process the following jobs on the machines shown, i. e. for each machine find the job which should be done first. Also, calculate the total time elapsed to complete both the jobs : **8**

| | | | | | | |
|---|---|---|---|---|---|---|
| Job 1 | sequence: | A | B | C | D | E |
| | time : | 3 | 4 | 2 | 6 | 2 |
| Job 2 | sequence: | B | C | A | D | E |
| | time : | 5 | 4 | 3 | 2 | 6 |

**OR**

c) A TV repairman finds that the time spent on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they come in, and if the arrival of sets is approximately Poisson with an average rate of 10 per 8-hour day, what is repairman's expected idle time each day? How many job are ahead of the average set just brought in? **8**

d) At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate 6 per hr and the railway station can be handle them as an average of 12 per hr. Assuming Poisson arrivals and exponential service distribution, find the steady state probability for the various number of trains in the system. Also find the average waiting time of a new train coming into the yard. **8**

## UNIT – IV

**4.** a) Obtain the set of necessary conditions for the N. L. P. P.: **8**

Maximize $z = kx^{-1}y^{-2}$ subject to constraints,

$x^2 + y^2 - a^2 = 0$ with $x \ge 0,\ y \ge 0$ &. hence find the minimum value of z.

b) Solve : Maximize $z = 3.6x_1 - 0.4x_1^2 + 1.6x_2 - 0.2x_2^2$ **8**

subject to the constraints : $2x_1 + x_2 \le 10$ and $x_1, x_2 \ge 0$

**OR**

c) Solve graphically the following NLPP: **8**

Maximize $z = 2x_1 + 3x_2$ subject to the constraints,

$x_1, x_2 \le 8,\ x_1^2 + x_2^2 \le 20$ and $x_1, x_2 \ge 0$.

Verify that the Kuhn-Tucker conditions hold for the maxima you obtain.

d) Use Wolfe's method to solve the QPP: **8**

Maximize $z = 2x_1 + 3x_2 - 2x_1^2$ subject to the constraints,

$x_1 + 4x_2 \le 4,\ x_1 + x_2 \le 2$ and $x_1, x_2 \ge 0$.

**5.** a) Explain the pure & mixed integer programming problems. **4**

b) Write the formulation of linear goal programming problem. **4**

c) State the characteristics of queueing system. **4**

d) Write the short note on 'general non – linear programming problem'. **4**

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